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NON-LINEAR EQUILIBRIUM CORRECTION IN US REAL MONEY BALANCES, 1869–1997

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ABSTRACT

Non-Linear Equilibrium Correction in US Real Money Balances, 1869–1997*

Several theoretical models of money demand imply non-linear functional forms for the aggregate demand for money characterized by smooth adjustment towards long-run equilibrium. In this Paper, we propose a non-linear equilibrium correction model of US money demand, which is shown to be stable over the sample period from 1869 to 1997.

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NON-TECHNICAL SUMMARY

Over the last decade or so, a large body of empirical research on modelling the demand for money has developed, mainly using cointegration and equilibrium correction techniques that allow joint characterization of the equilibrium level of aggregate money demand as well as the dynamic adjustment towards this equilibrium (see, *inter alia*, Hendry and Ericsson, 1991a,b; Baba, Hendry and Starr, 1992; Carlson, Hoffman, Keen and Rasche, 2000, and the references therein). One of the issues which has received widespread attention by researchers is concerned with the modelling of money demand using low-frequency (annual) data for relatively long sample periods.

Long samples, however, may potentially be inappropriate because of regime shifts, financial innovation and structural changes, requiring extreme care in testing and modelling. In this Paper, we propose an empirical model of US real money balances estimated using an updated version of the data set provided by Friedman and Schwartz (1982), spanning from 1869 through to 1997. Over the sample period examined, the monetary history of the US has seen a number of fundamental changes in exchange rate regimes, institutional structure and policy targets which, in addition to the continuous evolution of the financial system and various nominal and real shocks, represent serious potential pitfalls to researchers attempting to find an empirical model of money demand that is stable over the full sample. We address this task by employing a non-linear dynamic model for the change in US real money balances using econometric methods of the type popularized by Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998).

At a theoretical level, non-linearity in money demand adjustment is predicted by the class of target-bounds models developed by – *inter alios* – Miller and Orr (1966) and Akerlof (1973, 1979), where a representative agent specifies a target level of money balances and thresholds above and below the target which the balances must not cross, hence implying that short-run nominal adjustment occurs within bounds set by long-run magnitudes. At the macroeconomic level, this class of models is likely to generate smooth (as opposed to abrupt threshold) adjustment in the aggregate money demand function as an effect of time aggregation and non-synchronous adjustment by heterogeneous agents (Bertola and Caballero, 1990; Teräsvirta, 1994, 1998). Non-linear adjustment in money demand equations may similarly be rationalized on the basis of buffer stock models (e.g. Gandolfi and Lothian, 1976; Cuthbertson and Taylor, 1987), which recognize non-zero costs of adjustment of money balances and imply that it may be optimal for agents to allow short-run deviations of money balances from long-run equilibrium and to adjust only for relatively large deviations. In general, these types of models imply that the speed of adjustment in money demand functions in response to

exogenous shocks may depend non-linearly at the aggregate level on the size of the deviation from long-run equilibrium (for a discussion of these issues, see, for example, Milbourne, 1987, 1988; Thornton, 1990; Mizen, 1994, 1997; Sarno, 1999).

Our empirical results are encouraging on a number of fronts. We obtained a unique long-run money demand function relating real money, real income and the long-term interest rate, which displayed a plausible interest rate semi-elasticity. The dynamic model proposed is shown to be stable over the full sample period examined and the finding of significant non-linearity in the empirical money demand equation is consistent with several related empirical studies (e.g., *inter alia*, Hendry and Ericsson, 1991a; Lütkepohl, Teräsvirta and Wolters, 1999; Sarno, 1999; Teräsvirta and Eliasson, 2001).

Also, our dynamic model allows us to capture the predictions of the theoretical literature in the spirit of target-bounds models and buffer-stock models, being consistent with a world where the behaviour of fully optimizing agents who allow short-run deviations from the long-run equilibrium level of money balances generates smooth adjustment towards equilibrium in the aggregate money demand function.

Overall, our results suggest that failure to allow for non-linear dynamics characterized by short-run deviations from long-run equilibrium may contribute to explaining the difficulty of much empirical research in obtaining stable aggregate US money demand equations.

1. Introduction

Over the last decade or so, a large body of empirical research on modeling the demand for money has developed, mainly using cointegration and equilibrium correction techniques (see, *inter alia*, Hendry and Ericsson, 1991a,b; Baba, Hendry and Starr, 1992; Carlson, Hoffman, Keen and Rasche, 2000, and the references therein).[1] One of the issues which has received widespread attention by researchers is concerned with the dynamic modeling of money demand using low-frequency data for relatively long sample periods. Long samples, however, may potentially be inappropriate because of regime shifts, financial innovation and structural changes, requiring extreme care in testing and modeling. In this paper, we propose an empirical model of US real money balances estimated using an updated version of the data set provided by Friedman and Schwartz (1982), spanning from 1869 through to 1997.[2] Over the sample period examined, the monetary history of the US has seen a number of fundamental changes in exchange rate regimes, institutional structure and policy targets which, in addition to the continuous evolution of the financial system and various nominal and real shocks, represent serious potential pitfalls to researchers attempting to find an empirical model of money demand that is stable over the full sample. This task is addressed by

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[1] Several authors have recently begun to use the term 'equilibrium correction' instead of the traditional 'error correction' as the latter term now seems to have a different meaning in some recent theories of economic forecasting (e.g. see Clements and Hendry, 1998, p. 18). Since the term 'equilibrium correction' conveys the idea of the adjustment considered in the present context quite well, we use this term below.

[2] MacDonald and Taylor (1992) estimate a linear dynamic model for the US money demand function using the original Friedman-Schwartz data from 1869 to 1970 (for a similar approach applied to postwar data, see, for example, Rasche, 1987; Hoffman and Rasche, 1991; see also the earlier work by Thornton, 1982). Lucas (1988), Stock and Watson (1993) and Anderson and Rasche (1999) use similar data for somewhat longer sample periods. Nevertheless, the present paper represents, to the best of the authors' knowledge, the first attempt to model the US demand for money over a sample which extends the Friedman-Schwartz data set to cover the whole post-Bretton Woods period until the late 1990s.

employing a nonlinear equilibrium correction model for the change in US real money balances in the form of a smooth transition regression of the type popularized by Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998). The model proposed is shown to be stable over the full sample period examined and the finding of significant nonlinearity in the empirical money demand equation is consistent with several related empirical studies (e.g., *inter alia*, Hendry and Ericsson, 1991a; Lütkepohl, Teräsvirta and Wolters, 1999; Sarno, 1999; Teräsvirta and Eliasson, 2001).

At a theoretical level, nonlinearity in money demand adjustment is predicted by the class of target-bounds models developed by - *inter alios* - Miller and Orr (1966) and Akerlof (1973, 1979), where a representative agent specifies a target level of money balances and thresholds above and below the target which the balances must not cross, hence implying that short-run nominal adjustment occurs within bounds set by long-run magnitudes. At the macroeconomic level, this class of models is likely to generate smooth (as opposed to abrupt threshold) adjustment in the aggregate money demand function as an effect of time aggregation and nonsynchronous adjustment by heterogeneous agents (Bertola and Caballero, 1990; Teräsvirta, 1994, 1998). Nonlinear adjustment in money demand equations may similarly be rationalized on the basis of buffer stock models (e.g. Gandolfi and Lothian, 1976; Cuthbertson and Taylor, 1987), which recognize non-zero costs of adjustment of money balances and imply that it may be optimal for agents to allow short-run deviations of money balances from long-run equilibrium and to adjust only for relatively large deviations. In general, these types of models imply that the speed of adjustment in money demand functions in response to exogenous shocks may depend nonlinearly at the aggregate level on the size of the deviation from long-run equilibrium (for a discussion of these issues, see, for example, Milbourne, 1987, 1988; Thornton, 1990; Mizen, 1994, 1997; Sarno, 1999).[3]

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[3] While consistent with some of the implications of the buffer-stock money demand literature, however, it is perhaps fair to say that the empirical model proposed below does not aim to be a 'pure' test of the buffer-stock model, particularly because there is no attempt to model expectations (e.g. see Cuthbertson and Taylor, 1987; Sarno, 1999).

The nonlinear equilibrium correction model proposed in this paper allows us to capture the predictions of this strand of the theoretical literature, being consistent with a world where the behavior of fully optimizing agents who allow short-run deviations from the long-run equilibrium level of money balances generates smooth adjustment towards equilibrium in the aggregate money demand function.

The rest of the paper is structured as follows. In Section 2 we discuss the class of nonlinear models which we employ for modeling the demand for money and other aspects of our econometric methods. Section 3 describes our data set and reports the empirical results from carrying out unit root, cointegration and linearity tests as well as the results from applying linear and nonlinear equilibrium correction modeling techniques to our data. A final section briefly summarizes and concludes.

2. Nonlinear equilibrium correction modeling

In this paper, we consider a smooth transition regression (STR) model of money demand, parameterized in the form (Granger and Teräsvirta, 1993; Teräsvirta, 1998):

$$\Delta(m-p)_t = k_0 + \rho\hat{u}_{t-1} + \sum_{j=1}^{p}\alpha_j\Delta(m-p)_{t-j} + \sum_{j=0}^{p}\beta_j\Delta y_{t-j} +$$
$$\sum_{j=0}^{p}\gamma_j\Delta RL_{t-j} + \sum_{j=0}^{p}\delta_j\Delta RS_{t-j} +$$
$$[k_0^{'} + \rho^{'}\hat{u}_{t-1} + \sum_{j=1}^{p}\alpha_j^{'}\Delta(m-p)_{t-j} + \sum_{j=0}^{p}\beta_j^{'}\Delta y_{t-j} +$$
$$\sum_{j=0}^{p}\gamma_j^{'}\Delta RL_{t-j} + \sum_{j=0}^{p}\delta_j^{'}\Delta RS_{t-j}]\Phi[\theta;z_t-\mu]+\varepsilon_t, \tag{1}$$

where $\Delta x_t \equiv x_t - x_{t-1} \ \forall x$; m, p, y, RL and RS denote the logarithm of nominal money, the logarithm of the implicit price deflator (hence $m-p$ is the logarithm of real money), the logarithm of real income, the long- and short-term interest rates respectively; ε_t is white noise; and $\Phi[\cdot]$ is a parametric transition function with $\theta>0$ determining the speed of transition for a given level of $(z_t-\mu)$. In particular, we consider a STR model where the transition variable z_t is the lagged estimated equilibrium error from a conventional static long-run

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money demand model of the form $(m-p)_t = a + by_t + cRL_t + u_t$ - i.e. the lagged residuals $\hat{u}_{t-d}$, with the

integer $d>0$ denoting a delay parameter, so that $z_t \equiv \hat{u}_{t-d}$.[4]

The transition function $\Phi[\cdot]$ in equation (1) may be, for example, an exponential function of the

form $[1 - \exp\{-\theta\,(\hat{u}_{t-d} - \mu)^2\}]$ or a logistic function of the form $[1 + \exp\{-\theta\,(\hat{u}_{t-d} - \mu)\}]^{-1}$, resulting in an

exponential STR (ESTR) or a logistic STR (LSTR) respectively. The transition function of the LSTR is a

monotonically increasing function of $(\hat{u}_{t-d} - \mu)$ and yields asymmetric adjustment towards equilibrium,

whereas the transition function of the ESTR is symmetric about μ although the tendency to move back to

equilibrium is stronger the larger the absolute size of the deviation from equilibrium $\left|\,\hat{u}_{t-d} - \mu\,\right|$ (for further

details see Granger and Teräsvirta, 1993; Teräsvirta, 1994, 1998).

Given our discussion in Section 1, it is clear that the nonlinear adjustment described by target-

bounds models and buffer stock models for aggregate money demand functions, where the speed of

adjustment towards the long-run equilibrium depends on the absolute size of the deviation from equilibrium,

might be well captured by an exponential transition function.[5] Nevertheless, in selecting the transition

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[4] In this framework, the long-run demand for money is determined by income and the long-term interest

rate, although the *change* in the short-term interest rate is allowed to affect the dynamic adjustment towards

the long-run equilibrium. This has proved to be very effective in several cases in the present context (see,

inter alia, Hendry and Ericsson, 1991a; MacDonald and Taylor, 1992; Taylor, 1993). Nevertheless, some

authors have used the short-term interest rate (either in place of or in addition to the long-term rate) in the

long-run model; see, for example, the study by Hoffman, Rasche and Tieslau (1995) and the relevant

discussion on these issues in Laidler (1993) and Hoffman and Rasche (1996). Also, note that, as discussed

in Section 3, we also include four dummy variables in the long-run cointegrating money demand function

from which we retrieve the equilibrium error.

[5] The nonlinear ECM (1) is a direct generalization of the standard linear ECM employed in a vast literature

on money demand. The generalization obviously consists of the nonlinear component of the model, and the

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function in our empirical analysis we employ a purely statistical approach in that we execute the sequence of nested tests suggested by Granger and Teräsvirta (1993) and Teräsvirta (1998). Hence, as a preliminary to model specification and estimation, we executed linearity tests based on the auxiliary ordinary least squares regression:

$$\hat{v}_t^{ECM} = \zeta_0 \mathbf{W}_t + \zeta_1 \mathbf{W}_t \hat{u}_{t-d} + \zeta_2 \mathbf{W}_t \hat{u}_{t-d}^2 + \zeta_3 \mathbf{W}_t \hat{u}_{t-d}^3 + innovations, \tag{2}$$

where $\hat{v}_t^{ECM}$ denotes the residuals from a general linear equilibrium correction model (ECM) for $\Delta(m\text{-}p)_t$ as a function of $\mathbf{W}_t$, which is the vector comprising the explanatory variables $\Delta(m\text{-}p)_{t-1}$, $\Delta(m\text{-}p)_{t-2}$, Δy_{t-j}, ΔRL_{t-j} and ΔRS_{t-j} for j=0,1,2, in addition to a constant term and the lagged estimated cointegrating residuals or equilibrium correction term $\hat{u}_{t-1}$; ζ_0, ζ_1, ζ_2 and ζ_3 are vectors of parameters. A general test for linearity against STR-type nonlinearity is then the F-test of the null hypothesis H_{0G}: $\zeta_1 = \zeta_2 = \zeta_3 = \mathbf{0}$, where $\mathbf{0}$ is a null vector; H_{0G} can be tested for various values of d, say $d \in \{1,2,...,D\}$. If linearity is rejected for more than one value of d using such F-test (say F^G), then d is determined as the value $\hat{d}$ which minimizes the p-value of the linearity test and we set $d = \hat{d}$.

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variables included are standard variables included in money demand models previously estimated and reported in many of the studies cited in the paper. Also, given that the transition function $\Phi[\cdot]$ can, in principle, be any bounded nonlinear transition function, our nonlinear model is fairly general. Inevitably, however, we do need to restrict our attention to some specific parametric transition functions for which formal linearity tests exist, and this is one reason why we focus on the logistic and exponential functions. However, the exponential function seems economically plausible and fairly consistent with our theoretical considerations inspired by target-bounds models and buffer-stock models. The class of nonlinear models is infinite, and we have chosen to concentrate on the STR formulation primarily because of these attractive properties, its relative simplicity, and the large amount of previous research on the estimation of STR models.

After rejecting H_{0G} using F^G, the choice between LSTR and ESTR formulations may be based on a sequence of nested tests within H_{0G}. In practice, the following hypotheses are tested sequentially: H_{03}: $\zeta_3 = 0$, H_{02}: $\zeta_2 = 0 \mid \zeta_3 = 0$ and H_{01}: $\zeta_1 = 0 \mid \zeta_2 = \zeta_3 = 0$ using F-tests termed F^3, F^2 and F^1 respectively. If either F^3 or F^1 yields the strongest rejection of the linearity hypothesis (i.e. the lowest p-value), we select an LSTR model; if F^2 yields the strongest rejection of linearity, however, we select an ESTR model (for further details see Granger and Teräsvirta, 1993; Teräsvirta, 1998).

3. Data and empirical results

Annual time series for nominal narrow money, real income, implicit price deflator, short-term interest rate (call money rate in percent per annum) and long-term bond rate (percent per annum) were obtained from updating the data set provided by Friedman and Schwartz (1982) using the International Monetary Fund's *International Financial Statistics* CD. The sample period spans from 1869 to 1997. From these data we constructed the time series of interest for the empirical analysis, namely the logarithm of real money balances (m-p), the logarithm of real income (y), the long-term interest rate (RL) and the short-term interest rate (RS).

As a preliminary exercise, we tested for unit root behavior of each of (m-p), y, RL and RS by calculating augmented Dickey-Fuller (ADF) test statistics (Fuller, 1976; Dickey and Fuller, 1979). In each case, the number of lags was chosen such that no residual autocorrelation was evident in the auxiliary regressions. In keeping with a large number of studies in this context, we were in each case unable to reject the unit root null hypothesis at conventional nominal levels of significance. On the other hand, putting the series into first-difference form did appear to induce stationarity for each of the series. These results confirm the often-recorded result that real money balances, real income and interest rates are generated by processes with a single unit root (e.g. see, *inter alia*, Nelson and Plosser, 1982; Hendry and Ericsson, 1991a,b; Baba, Hendry and Starr, 1992; Carlson, Hoffman, Keen and Rasche, 2000, and the references therein). However, the possibility that the time series in question display structural breaks complicates the interpretation of unit root

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tests. Thus, we constructed four constant shift dummies, say D1, D2, D3 and D4, covering the first world

war (1914-1918), the interwar period (1919-1938), the second world war (1939-1945), and the postwar

period (1946-1997) respectively. We then re-calculated the ADF tests using an auxiliary regression that also

included the four dummies, essentially using a variant of the procedure employed by Perron (1989).

Nevertheless, the results from these unit root tests yielded qualitatively identical results to the conventional

ADF tests in that we were unable to reject the unit root null hypothesis for each of $(m-p)$, y, RL and RS.[6]

Given our unit root tests results, we then formally tested for cointegration using the Johansen (1988,

1995) maximum likelihood procedure in a vector autoregression comprising $(m-p)$, y, RL, an unrestricted

intercept term and the four constant shift dummies D1, D2, D3 and D4 defined above. We assumed a lag

length of two, which was suggested by both the Akaike information criterion and the Schwartz information

criterion. Using both Johansen test statistics (namely the λ_{max} and λ_{trace} statistics) and the appropriate critical

values (both asymptotic and corrected to adjust for finite sample and for the presence of the four dummies

described above), the cointegration results suggested that one cointegrating vector exists between $(m-p)$, y

and RL; after normalizing the coefficient on $(m-p)$ to minus unity, the estimated cointegrating parameters on

y and RL were found to be very strongly statistically significantly different from zero at conventional

nominal levels of significance and equal to $\hat{b} = 1.441$ and $\hat{c} = -0.058$ respectively.[7] These results are fairly

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[6] Moreover, using non-augmented Dickey Fuller tests or augmented Dickey-Fuller tests with any number of

lags in the range between 1 and 5 yielded qualitatively identical results, regardless of whether the auxiliary

regression included a constant or a constant and a deterministic time trend and of whether it included

dummies. The results from executing all of these preliminary unit root tests are not reported to conserve

space.

[7] Balke and Fomby (1997) show that the Johansen method of estimating the cointegrating vector may work

reasonably well in the presence of nonlinear threshold cointegration, and conjecture that these results may

also hold for smooth transition nonlinearities in adjustment - see also Corradi, Swanson and White (2000).

We also employed the procedure suggested by Phillips and Hansen (1990) to test for cointegration in our

satisfactory in that the cointegrating parameters are correctly signed, although it is disappointing that the hypothesis of income homogeneity ($\hat{b}$ =1) is rejected at conventional nominal levels of significance.[8] We then retrieved the cointegrating residuals, $\hat{u}_t$, which we use to estimate an ECM and which we also consider as the potential transition variable in our nonlinear equilibrium correction modeling.

However, before proceeding to estimating an ECM for the change in real money balances, we tested whether the cointegrating equation describing the long-run demand for money has been stable over our sample period. A number of tests have been developed for testing whether long-run cointegrating relationships display structural breaks (see, for example, Hansen, 1992; Andrews, 1993; Gregory and Hansen, 1996). In this paper, we use the tests suggested by Hansen (1992). Given a static model, Hansen proposes three tests of the null hypothesis that the cointegrating vector, say ϑ_t, is constant over the sample period (T). All three tests are based on a sequence of *F*-tests, say $F_{T,t}$ for 1<t<T. The first test proposed by Hansen (1992) tests the null of parameter stability against the alternative hypothesis that there is a single break-point at time s (i.e. H$_1$: $\vartheta_{1,t} \neq \vartheta_{s+1,T}$ where $\vartheta_{i,j}$ indicates the value of the cointegrating vector over the interval t=i,…,j $\forall$i,j such that i<j). The problem with this test is that the break point s is treated as known (see Hansen, 1992). A second test proposed by Hansen (1992), the $\sup(F)$ test, treats the break point as unknown so that the alternative hypothesis is now H$_1$: $\vartheta_{1,t} \neq \vartheta_{\tau+1,T}$ where $\tau = (t/T) \in \Im$ and $\Im$ is a compact subset of (0,1). The test is computed as $\sup(F) = \sup_t (F_{T,t})$. The third test proposed by Hansen (1992)

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long-run money demand model and found estimates of the cointegrating parameters identical to the ones reported above up to the second decimal point.

[8] Under the assumption that the rank of the long-run matrix of the cointegrating vector autoregression equals unity, we could not reject the hypothesis of weak exogeneity of *y* and *RL* in the static long-run model. Establishing weak exogeneity of the regressors in the long-run money demand model allows us to model real money balances within a single-equation equilibrium-correction framework (Engle, Hendry and Richard, 1983).

considers the possibility that the cointegrating vector ϑ_t follows a martingale process, so that there is a constant hazard of parameter instability, and H$_1$: $\vartheta_t = \vartheta_{t-1}$ + error. This test may be computed as *mean* $(F) =$ $(1/T^*) \sum_{\tau \in \Im}^{T} F_{T,t}$, and T$^* = \sum_{\tau \in \Im}^{T} 1$. The tests are applied on a region that does not include the end-points of the sample, and Hansen (1992) and Andrews (1993) suggest a trimming region $\Im = [0.15, 0.85]$, which in our case corresponds to the subperiod 1888-1978. The results from carrying out the Hansen (1992) sup (F) and *mean* (F) tests suggest that the null hypothesis of constancy of the cointegrating vector over the sample could not be rejected, with *p*-values of 0.32 and 0.30 for the sup (F) and *mean* (F) tests respectively.[9] In turn, these results indicate that the long-run money demand function implied by our cointegration results is stable over the sample period examined even when the null of parameter stability is tested against alternative hypotheses of instability with unknown break points.

Prior to testing for linearity we estimated a linear ECM for $\Delta(m\text{-}p)_t$ with a lag length of two and tested down by sequentially imposing restrictions on statistically insignificant parameters in order to obtain the best fitting parsimonious model, reported in Table 1. The model appears to be quite adequate in terms of fit and displays approximately white noise residuals. Also, each of the estimated parameters is of an economically plausible sign and magnitude. Nevertheless, the linear ECM does not pass the diagnostic test for parameter stability - the null of parameter stability against the alternative of smoothly changing parameters (see Lin and Teräsvirta, 1994; Eithreim and Teräsvirta, 1996; Teräsvirta, 1998; Wolters, Teräsvirta and Lütkepohl, 1998) was rejected very strongly. Also, the linear ECM does not pass the RESET test at conventional significance levels. Clearly, this may be interpreted as suggestive of the fact that nonlinear equilibrium correction may be a prerequisite for parameter stability, consistent with the evidence

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[9] The $F_{T,t}$ test is asymptotically distributed as χ^2 under the null hypothesis. The asymptotic critical values for the sup (F) and *mean* (F) tests, computed by Monte Carlo simulations, are given in Hansen (1992). The Hansen (1992) stability tests reported here were obtained using the GAUSS programme FM.PRG available at Hansen's website (http://www.ssc.wisc.edu/~bhansen).

recently reported on other long spans of data by Sarno (1999) for Italy and Teräsvirta and Eliasson (2001) for the UK.

Next, applying the Akaike and Schwartz information criteria to a linear ECM for $\Delta(m\text{-}p)_t$, the lag length p was set equal to two in order to execute the linearity tests discussed in Section 2. Panel A of Table 2 reports p-values of the test statistic F^G for $d \in \{1,2,...,5\}$. Linearity is rejected most strongly for $d=1$, suggesting a rather fast response of real money balances to deviations from its linear equilibrium path. Panel B of Table 2 then reports the p-values of the test statistics F^3, F^2 and F^1 assuming $d=1$; the results clearly suggest that an ESTR is the model indicated by the procedure developed by Granger and Teräsvirta (1993) and Teräsvirta (1998), consistent with our economic priors and the theoretical considerations discussed in Section 1.[10] [11]

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[10] The choice of the transition variable was initially not restricted to the lagged equilibrium error, in that we also experimented with a number of other potential transition variables, including lagged changes in real money balances, income and both short- and long-term interest rates. However, linearity tests with the lagged equilibrium error as the transition variable yielded the strongest rejections of the null of linearity and, hence, we only report these results in the empirical analysis to conserve space. Also, note that using the general linear ECM described above to obtain the residuals tested for linearity or using the best linear ECM given in Table 1 below yielded qualitatively identical results.

[11] We addressed thoroughly the question of the robustness of our linearity tests results. The main concern involves the possibility of a spurious rejection of the linearity hypothesis under the test statistic F^G in equation (2) in finite sample. We addressed this issue by executing a number of Monte Carlo experiments constructed using 5,000 replications in each experiment, and with identical random numbers across experiments. Our simulations, based on a sample size comprising $T \in \{50,75,100,125,150,175,200\}$ artificial data points, suggested that the linearity test F^G does not tend to over-reject the null hypothesis of linearity when the true data generating process is linear autoregressive - *even if nonstationary* - and rejection of the null does not occur by chance when the process is even marginally nonlinear, thus increasing our confidence

Assuming p=2 and d=1 according to the linearity tests results, we then estimated an ESTR model of the form (1) by nonlinear least squares (Granger and Teräsvirta, 1993). We also followed the recommendation of Granger and Teräsvirta (1993) and Teräsvirta (1998) of standardizing the transition parameter by dividing it by the sample variance of the transition variable, $\hat{\sigma}_u$ and using a starting value of θ=1 in the estimation algorithm. A parsimonious nonlinear ECM was obtained for the change in US real money balances after imposing various exclusion restrictions (see the LR test in Table 3) in applying the conventional general-to-specific procedure (e.g. see Hendry, Pagan and Sargan, 1984). The resulting model displays insignificant diagnostics (including the appropriate parameter constancy test), yielding a sizable reduction in the residual variance relative to the best fitting linear ECM (about 15 percent). Nevertheless, using the minimal nesting strategy of Mizon and Richard (1986) and applying a simplification encompassing test between the STR model given in Table 3 and the best fitting linear ECM in Table 1 as done, for example, in Sarno (1999) and Teräsvirta and Eliasson (2001), indicated that the nonlinear ECM does not encompass the linear ECM at the five percent significance level (although it does at the one percent level) without being encompassed.[12][13]

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in the linearity tests results and, hence, on the validity of the nonlinear ECM proposed below. These results are interesting in that it appears that nonstationarity does not affect the size of the linearity tests (full details on the Monte Carlo simulations are available on request).

[12] More precisely, the null hypothesis that the STR model in Table 3 encompasses the best fitting linear ECM was rejected with a *p*-value of 0.0281, while the null that the best fitting linear ECM encompasses the STR model in Table 3 was strongly rejected with a *p*-value of 4.00×10^{-8}.

[13] Note that the effective importance of the lagged dependent variable (i.e. lagged real money balances) shrinks with the size of the deviation from equilibrium in our estimated STR model. This is interesting because a number of authors have argued that the sluggish adjustment implied by a statistically significant lagged dependent variable is hard to rationalize at a theoretical level (see e.g. McCallum and Goodfriend, 1988; Laidler, 1990; Goodfriend, 1985; Taylor, 1994). The implication in the present context is that inertia

The strongly nonlinear behavior implied by our empirical model is made clear by the plot of the estimated transition function against the transition variable $\hat{u}_{t-1}$, displayed in Panel a) of Figure 1. Note that the exponential transition function is bounded between zero and unity, $\Phi[\cdot]: \Re \rightarrow [0,1]$, has the properties $\Phi[0]=0$ and $\lim_{x \rightarrow \pm\infty} \Phi[x] = 1$, and is symmetrically inverse-bell shaped around zero. These properties of the ESTR model are attractive in the present modeling context because they allow a smooth transition between regimes and symmetric adjustment of real money balances for deviations above and below the long-run equilibrium level. The plot shows that the limiting case of the transition function $\Phi[\cdot]=1$ is attained, which indicates that the speed of transition between regimes is, in fact, very fast for large deviations from long-run equilibrium. The satisfactory goodness of fit of the nonlinear ECM is then highlighted by Panel b) of Figure 1, which plots actual and fitted values of changes in real money balances over the sample and shows that the fitted values are reasonably close to the actual values.[14]

4. Conclusion

A stylized fact in the empirical literature on modeling US money demand is the difficulty of obtaining stable empirical equations over relatively long sample periods. In this paper we have applied recently

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is reduced as the size of the disequilibrium grows, consistent with the buffer-stock and targets-bounds money demand models discussed above.

[14] One might wish to allow for the possibility that the nonlinearity was in fact of the threshold variety, which would also be consistent with target-bounds money demand models under certain aggregation conditions. Therefore, we considered a related parameterization of the STR model with a transition function that allows for threshold-type nonlinearity (Jansen and Teräsvirta, 1996 and Teräsvirta, 1998), so that the corresponding STR model then becomes a special case of a threshold equilibrium correction model (TECM). In our attempts to estimate a model of this kind on our data, however, we experienced severe problems in achieving convergence of the estimation algorithm, which appears to suggest that smooth rather than discrete adjustment in regime may be more appropriate on our aggregate data.

developed nonlinear econometric techniques to an updated version of the Friedman-Schwartz data set in order to model the demand for money in the US during the period 1869-1997. The results are encouraging on a number of fronts. We obtained a unique long-run money demand function relating real money, real income and the long-term interest rate, which displayed a plausible interest rate semi-elasticity of -0.058. Also, a dynamic evolution equation for the change in real money balances was obtained by estimating a nonlinear equilibrium correction in the form of an exponential smooth transition regression with the lagged long-run equilibrium error acting as the transition variable, implying faster adjustment towards equilibrium the greater the absolute size of the deviation from equilibrium.

While our results aid our understanding of nonlinear dynamics in the demand for money, we view them as a tentatively adequate characterization of the data, but which may be capable of improvement. Nevertheless, the nonlinear empirical money demand equation proposed here yields a sizable reduction in the residual variance relative to the best fitting linear equilibrium correction model of about 15 percent and appears to be superior to linear money demand modeling in several respects, also passing a battery of diagnostic tests and displaying parameter constancy despite the number of fundamental changes characterizing the monetary history of the US over our long sample period. Further, the empirical model proposed has a fairly natural interpretation in the light of the nonlinear type of adjustment implied by target-bounds models and buffer stock models of money demand. Overall, our results suggest that failure to allow for nonlinear dynamics characterized by short-run deviations from long-run equilibrium may contribute to explaining the difficulty of much empirical research in obtaining stable aggregate US money demand equations.

Table 1. Estimated linear ECM for the change in US real money balances

$$\Delta(m-p)_t = h_0 + h_1 \Delta(m-p)_{t-1} + h_2 \Delta y_t +$$
$$h_3 \Delta RS_t + h_4 \Delta RS_{t-1} + h_5 D1 + h_6 D4 + h_7 \hat{u}_{t-1}$$

where:

$h_0 = 0.046$ (0.007), $h_1 = 0.162$ (0.078), $h_2 = 0.277$ (0.060), $h_3 = -0.004$ (0.001),

$h_4 = -0.005$ (0.001), $h_5 = -0.039$ (0.018), $h_6 = -0.049$ (0.010), $h_7 = -0.075$ (0.021).

$\overline{R}^2 = 0.340$; DW = 2.017; LB = {0.409}; ARCH = {0.648}; RESET = {5E-06}

LM3 = {0.0035}; LM2 = {4E-0.6}; LM1 = {0.0156}

Notes: Estimation is by ordinary least squares; figures in parentheses are estimated standard errors. $\overline{R}^2$ is the adjusted coefficient of determination; DW is the Durbin-Watson statistic; LB and ARCH are the Ljung-Box test for residual autocorrelation up to order three and a test for autoregressive conditional heteroskedasticity up to order three respectively. RESET is the Ramsey (1969) test of the null hypothesis of linearity against the alternative hypothesis of general model misspecification, where the alternative model considered involves a second-order polynomial. LM3, LM2 and LM1 are tests of the null hypothesis that all coefficients except the coefficients of the dummy variables are constant against the alternative hypothesis that they are smoothly changing parameters, constructed as suggested by Lin and Teräsvirta (1994). For LB, ARCH and RESET as well as for LM3, LM2 and LM1 we only report the *p*-value in braces.

Table 2. Linearity tests results: *p*-values

Panel A: F^G tests

d=1: 1E-04;   d=2: 0.0446;   d=3: 1910;   d=4: 0.0145;   d=5: 0.2449

Panel B: F^3, F^2 and F^1 tests (d=1)

F^3: 0.0240; F^2: 3E-04; F^1: 0.2660

Notes: The F^G, F^3, F^2 and F^1 statistics are linearity tests constructed as described in the text; *d* denotes the delay parameter. The *p*-values reported above were calculated using the appropriate *F*-distribution.

Table 3. Estimated nonlinear ECM for the change in US real money balances

$$\Delta(m-p)_t = k_0 + \gamma_0 \Delta RL_t + [\alpha_1^{'} \Delta(m-p)_{t-1} +$$
$$\beta_0^{'} \Delta y_t + \tau_1^{'} D1 + \tau_4^{'} D4 + \rho^{'} \hat{u}_{t-1}][1 - \exp\{-(\theta/\hat{\sigma}_u)\hat{u}_{t-1}^2\}]$$

where:

$k_0 = 0.048$ (4.90E-03), $\gamma_0 = -0.012$ (5.23E-03), $\alpha_1^{'} = 0.394$ (0.143), $\beta_0^{'} = 0.537$ (0.130),

$\tau_1^{'} = -0.047$ (0.016), $\tau_4^{'} = -0.103$ (0.023), $\rho^{'} = -0.141$ (0.032), $\theta = 0.889$ (0.362), $\hat{\sigma}_u = 0.054$.

$\overline{R}^2 = 0.434$; V = 0.857; PC = {0.359}; NRN = {0.641};

NSC = {0.447}; ARCH = {0.825}; LR = {0.466}

Notes: Estimation is by nonlinear least squares; figures in parentheses are estimated standard errors. $\overline{R}^2$ is the adjusted coefficient of determination; V is the ratio of the estimated residual variance from the ESTR model to the estimated residual variance from the best fitting linear ECM. PC and NRN are Lagrange Multiplier-type tests for parameter constancy and for no remaining nonlinearity respectively, while NSC is a Lagrange Multiplier test for no serial correlation up to order three, constructed as suggested in Eitrheim and Teräsvirta (1996). ARCH is a test for autoregressive conditional heteroskedasticity up to order three. LR is the likelihood ratio test for the restrictions imposed in the estimated ESTR model against a general unrestricted ESTR model with $p=2$ and $d=1$ and is distributed as $\chi^2(q)$ with q equal to the number of restrictions. For each of PC, NRN, NSC, ARCH and LR we only report the p-value in braces.

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Figure 1: Nonlinear ECM for the change in real money balances

a) Estimated transition function
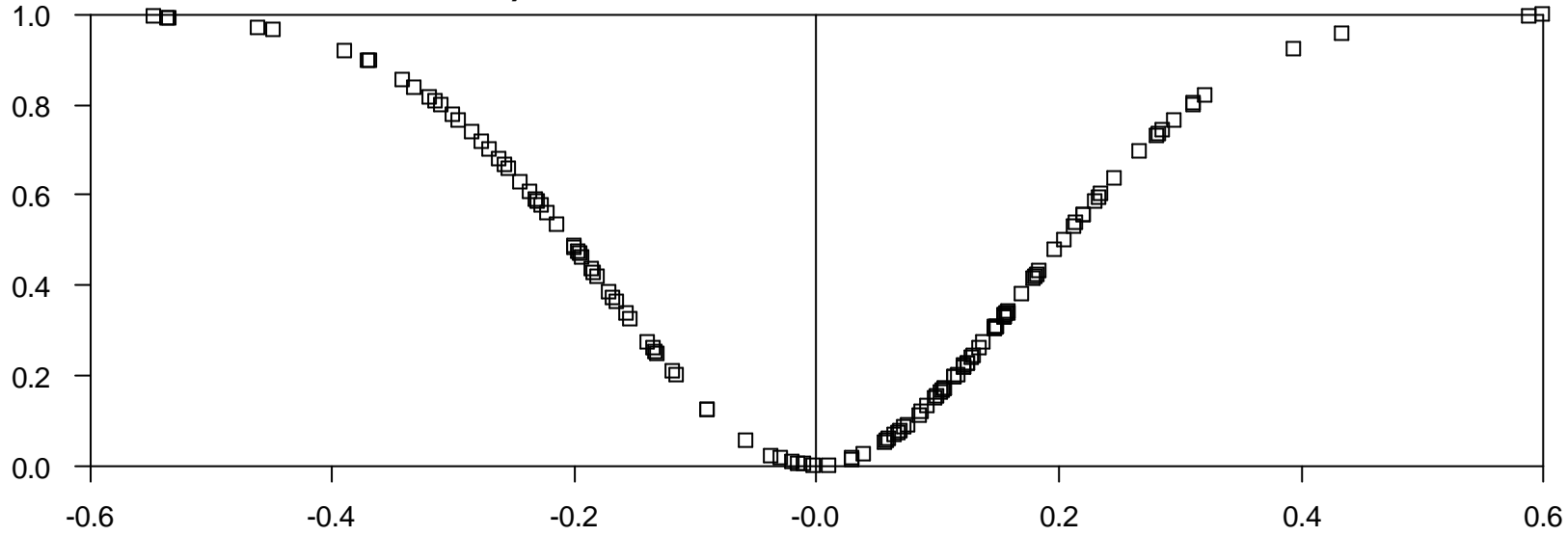
b) Actual (solid line) and fitted (dashed line) values